

Make Best Decisions Through Operations Research

Operations Research

THEORY, METHODS AND APPLICATIONS

FOR

Honours, Post-Graduate Degree and Diploma Courses of Statistics, Mathematics, Engineering,
Business Management, Operational Management, Marketing & Sales Management,
Computer Science, Commerce, Economics, C.A., I.C.W.A., I.S.I., M.B.A.,
M.C.A., I.I.Sc., Indian Civil Services (I.A.S.),
Indian Engineering Services (I.E.S.),
NET/SLET and Other Competitive
Examinations.

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- ☛ Since the subject is quite new to the students, each topic has been made clear through a sufficient number of solved examples.
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- ☛ Up-to-date year-wise references of various universities examination-papers are also given throughout the book.
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—S.D. SHARMA

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PREFACE

TO THE FIFTEENTH EDITION

It gives me pleasure to inform you that the present edition on '**Operations Research**' is now improved, well-organised and made up-to-date in the light of suggestions received from the learned teachers, students and professionals from various *Universities, Institutes and Organisations* in India and abroad. Besides the improvement in its presentation, the following major changes have been made in this new edition :

- * Errors and omissions are corrected.
- * A large number of solved and unsolved problems of *various examinations* are added.
- * Sufficient *hints* and *answers* are given along with *the examination problems*.
- * In most of the chapters, some *new topics* have been incorporated.
- * *Objective Questions* are also given at the end of each chapter.

In view of above large scale improvements, it can be assured that this new edition will definitely raise the horizon of the knowledge in the subject of *Operations Research*. It has grown out of the lectures given to *post-graduate students* during the past 37 years in *India and abroad*.

Acknowledgements : I am particularly grateful to Dr. James Glim (Chair and Distinguished Professor, Deptt. of Mathematics and Statistics, SUNY, USA); Prof. B. D. Sharma (Professor, Deptt. of Math. Sci., Clark, Atlanta University, U.S.A.); Dr. D.S. Tomer (Deptt. of Computer Science, University of Arkansa, U.S.A); Prof. C. Mohan, Prof. Bani Singh, Prof. H. G. Sharma (Deptt. of Maths, IIT Roorkee); Prof. G. C. Sharma (Head of Maths, Deptt., Agra University), Prof. B. B. Sharma (Deptt. of Maths, Himachal Pradesh University), Prof. N. K. Malhotra (Secretary, ORSI, Delhi Chapter, New Delhi), Prof. C. P. Bajaj (Deptt. of OR, Delhi University), Dr. D.K. Sharma and Dr. Julius A. Alade (University of Maryland, U.S.A.), Dr. R. P. Singh (Head of Maths. Deptt., L. R. College, Sahibabad (Gzd.)), Dr. S. N. Gupta & Dr. S. M. Gupta (Deptt. of Statistics, Meerut College), Dr. G. Ravindra Babu (Head, Deptt. of Science & Humanities, Mahavir Inst. of Science & Technology, Hyderabad), Dr. Rajeev Jain (Deptt. of Maths. D. A. V. College, Dehradun), Dr. Sanjeev Rajan (Deptt. of Maths, Hindu College, Moradabad), Dr. A.K. Bansal (Computer Programmer and Systems Analyst, UCMS, New Delhi, Mr. Vikash Saxena (Dewan Mgt. College, Meerut) for their generous help in several ways time to time.

Also, I am highly thankful to Prof. P. Mahabaleswarappa (Head, Deptt. of Mech. Engg., S.J.M. Institute of Technology, Chitradurga, Karnataka) for motivating the students of VTU about the book on this subject.

I am also thankful to the Publishers, Printers, and Vibgyor for its improvement during the type-setting process.

Suggestions for further improvement of the book will be highly appreciated and thankfully acknowledged.

D-215, Shastri Nagar
Meerut—(India)

—S.D. SHARMA

Dear Students,

May be your friends have recently appeared in any examination of *operations research*. We shall be grateful if you could send us the question-papers and the syllabi of your course in this subject. The original papers will be returned soon after the use at your address.

—S.D. SHARMA

TO THE FIRST EDITION

It is indeed a very heartening situation that the subject of '*Operations Research*' has been growing theoretically and in its applications to a wide variety of problems in diverse fields of life, namely : *engineering, business, management, economics, and medical sciences* etc. In view of this, *Operations Research* needs to be taught as a part of *Job-Oriented Courses*.

In the context of tremendous pace at which *engineering* and *technology* is advancing and imposing new demands on the *research & development* activities, the scientist or practising engineer who has to interpret science to practical end has the obligation to keep himself ever alert to understand the implications and complexities of science and engineering before he can utilise them to the benefits of his fellowmen. The subject matter covered in this book not only brings better awareness but opens newer possibilities for greater use of these powerful tools (in *planning, scheduling, cost and job control*) to the efficient and economical conduct of industrial endeavours for which optimum use of *men, money, machines & materials*, and their management at all levels is inescapable.

The main aim of this book is to make it clear the fundamentals of *Operations Research* and its techniques used in different fields of interest. The present form of the book is the reservoir of knowledge and experience readily made available. However, in order to maintain continuity and harmonize the related topics dealt with and also to clarify many points which could not be made available in print, the author has taken the liberty to enlarge, modify and provide fill-ups wherever necessary. The author has also quoted practical problems as far as possible.

It is hoped that this book will be useful for the students as well as for the professionals in their respective fields who wish to have indepth study of these topics. Students of *Mathematics, Statistics, Operations Research, Engineering, Economics, and Management* etc., whose courses of study include some or all of the topics reported in this book will be benefited.

I am particularly grateful to Prof. P.S. Iyar and Smt. N.A. Savitri for their generous help in pointing out errors and omissions, and Shri A.K. Gupta (Technical Officer, Calcutta) for his editorial assistance. I am highly indebted to Prof. Banwari Lal and Dr. L.R. Goel for their great help in writing this book. I also take this opportunity to express my sincere gratitude to Dr. R.K. Gupta (Professor of Mathematics) who gave me the inspiration throughout the preparation of this book. My grateful thanks are also due to Dr. K.S. Shah (Professor of Mechanical Engg.) for his fruitful discussions in making the book useful for *Engineering Students*. I am also thankful to Mr. Basab Ray for taking much pains and keen interest in constantly writing me his remarkable suggestions.

In the end, I wish to thank the Publishers and the Printers for their full cooperation in bringing out the book in the present nice form.

Suggestions for further improvement of the book will be gratefully acknowledged.

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—S.D. SHARMA

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